Proceeding of the 11th World Congress on Intelligent Control and Automation
Shenyang, China, June 29 - July 4 2014

Fuzzy-Logic Based Adaptive Weighting Filter for Strap-Down Inertial Navigation Systems *

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***Abstract*— Strap-down inertial navigation systems are widely applicable in both military and civil industries. The outputs of gyroscope and accelerometer are polluted by noises. Different form the conventional technologies, the weighting filter presented in this work is superior to both the complementary filter and Kalman filter. Furthermore, to eliminate the negative effects brought by the variation of noise statistical characteristics, a fuzzy-logic based regulator is introduced to adjust the weight coefficient adaptively. Finally, experiments are conducted to validate the efficiency of the filter.**

***Index Terms*— Adaptive weighting filter, Fuzzy-logic, Strap-down inertial navigation system, Complementary filter, Kalman filter.**

I. Introduction

The strap-down inertial navigation system (SINS), composed of inertial measurement units (gyroscope and accelerometer) mounted on the aircraft, naval-craft and missile that require navigation information such as attitude, velocity, orientation, is a navigation technology transforming the detected signals to navigation information. Different from the platform inertial navigation system (PINS), which is complex, expensive, bulky and heavy, the SINS which is absences of inertial platform, works without the external reference to behave as an autonomous navigation system, being more and more often applied in military and civil industries [8].

The SINS is usually employed to determine the object attitude, namely, the rotation angle on two degree-of-freedom, the pitch and roll. It can be estimated by the integration of gyroscope output over time. The gyroscope, however, suffers from high drift and noise where the estimate will deteriorate over time. On the other hand, measurement using accelerometer under the static situation is accurate. However, due to the absence of ability to distinguish the gravitational and non-gravitational acceleration, the rotation angle detected by accelerometer diverges seriously for the presence of motion acceleration [3], [11].

To overcome these problems, a common method is to employ the multi-sensor fusion technology. Due to the complementary characteristics of gyroscope and accelerometer, the complementary filter, complementary in the frequency domain, performs a frequency fusion of the two sensors without considering the noise statistical characteristics [3], [7]. Although there is no guarantee of optimality, this approach is cost-efficient and industrially reliable [7]. As a model-based approach, the Kalman filter behaves as a optimal estimator under the case where process and measurement noises are two uncorrelated Gaussian white noises [14]. Furthermore, some researchers introduced global position system (GPS) to increase the accuracy, and remove the non-gravitational acceleration from accelerometer with the assist of GPS. To enable the indoor navigation where GPS is unavailable, wireless fidelity is introduced to establish the so-called INS/Wi-Fi [4]–[6]. More sensors bring higher accuracy, coupled with a great cost.

However, more and more researchers have realized that the noise statistical characteristics variation is the primary factor that affects the filtering accuracy. So the development of adaptive filter is being paid more and more attentions. A traditional approach is to estimate the noise statistical characteristics based on sampled data using maximum a posteriori estimator (MAPE), but this method is computationally complex and requires the distribution of noise statistical

*Corresponding author: Yu Kang. This work was supported in part by the National Natural Science Foundation of China under Grant 61174061, 61304048 and 61174045; the Program for New Century Excellent Talents in University under Grant NCET-10-0985 and NCET-12-0195; the Foundation of Key Laboratory of Autonomous Systems and Networked Control (Chinese Ministry of Education) under Grant 2013A04; the Fundamental Research Funds for the Central Universities under Grant 2013ZG0035; and the Ph.D. Programs Foundation of Ministry of Education of China under Grant 20130172110026, and also gratefully acknowledges supports from the Youth Innovation Promotion Association, Chinese Academy of Sciences.

978-1-4799-5825-2/14/$31.00 ©2014 IEEE

characteristics to be known first [2]. With a number of experiments being conducted, we found that there exists a relationship between the sampled data and noise statistical characteristics. Not easy to establish the precise mathematical relationship, the fuzzy-logic is employed to solve this problem [5], [12].

The rest of paper is structured as follows. Section 2 presents the description of the hardware system and the related mathematical model. Section 3 states the problems to be solved coupled with the noise statistical characteristics obtained by experiments. Section 4 exhibits the design of fuzzy-logic based adaptive weighting filter. Section 5 shows the experiment results. This paper is concluded in Section 6.

II. System Description

The SINS mainly fuses the data from gyroscope and accelerometer to obtain relative accurate navigation information. To validate its accuracy, a more accurate device should be sought as the reference. The mechanical gyroscope with gimbaled frame, bulky and expensive but accurate, is one of possible references [8]. Another one designed a pendulum formed by a absolute encoder, which is extremely high in accuracy [13]. This scheme is low in cost and easy to implement, but is able to measure the rotation angle on one degree-of-freedom only, the roll or the pitch. However, it is noted that the roll and pitch angle measurement are independent and share the same approach, namely, The method to estimate any of them can be applied on another without losing accuracy. To reduce the cost and facilitate the implementation, the second scheme is adopted and successfully manufactured as shown in Fig. 1.

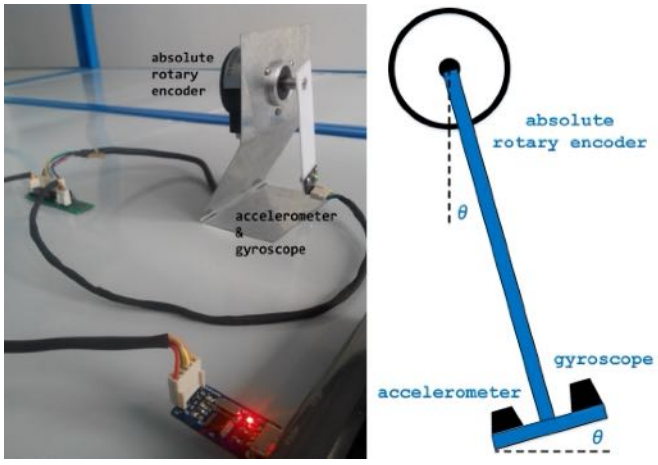


Fig. 1. Photograph and illustration of experiment device.

As shown in the figure above, the device mainly consists of a absolute encoder (KOYO TRD-NA1024NW, 1024 pulses), gyroscope and accelerometer (MPU-6050, integrated gyroscope and accelerometer on one chip, accuracy: 0.0076 deg/s and 0.00006 g, respectively) as marked. The other portions are auxiliary units, data-collector (STC-12C4052AD integrated circuit), communication port (TTL to serial port convertor) and computer (an ordinary PC). The rotation point of pendulum is mounted on the shaft of absolute encoder to transform rotation angle form the pendulum to absolute encoder. The gyroscope and accelerometer are mounted on the other end of pendulum parallel to ground. As known from illustration, the rotation angle of pendulum is equal to the inclination angle of the sensors, acquiring a relatively accurate reference by absolute encoder. It is noted that, due to that the device can not be manufactured precisely, a sampling at the static state is necessary to provide the reference point. The motion of pendulum can be driven by a DC motor or some other method. As shown in Fig. 2, the data-collector collects data form gyroscope, accelerometer and absolute encoder every 30 milliseconds, then the sampled data are sent to computer via communication port, fusing and displaying. The data-collector is controlled by computer on when and how to collect.

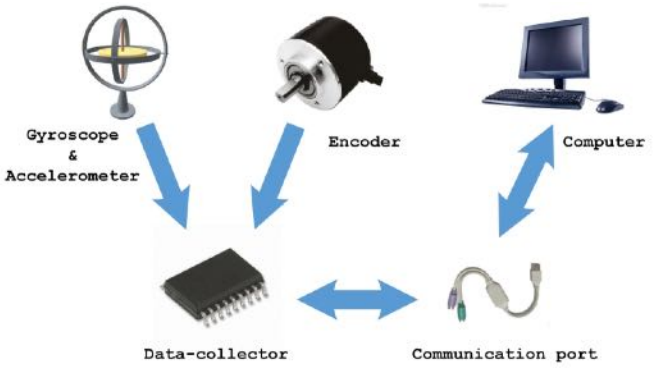


Fig. 2. Architecture of experiment device.

Considering the noises, the mathematical model, predictor equation and corrector equation are governed by the linear stochastic difference equations as follows.

$$\begin{aligned} x(k) &= x(k-1) + \tau u(k-1) + \eta(k-1) \\ z(k) &= x(k+v(k) \end{aligned} \tag{1}$$

where $x(k)$ is the real rotation angle at sampling time step k, $u(k)$ the output of gyroscope at sampling time step k, $z(k)$ the output of accelerometer at sampling time step k, τ the sampling interval. $\eta(k)$ and $v(k)$ are process and measurement noises, respectively.

III. Problem Statement

The noise statistical characteristics are unknown yet, so experiments on the device shown in Fig. 1 are conducted to obtain some a priori knowledge. The results are shown in Fig. 5 and Fig. 6.

As shown in Fig. 5 and Fig. 6, The red curves represent for reference, blue curves for the rotation angle estimate from the

the integration of gyroscope output and accelerometer output, respectively. The green curves are the differences between reference and estimate, namely the noises. The black curve in Fig. 6 shows the accumulation error from the bias of gyroscope. It is easy to find that the noise from gyroscope can be analogously treated as Gaussian noise with an extreme small mean. This non-zero mean caused by bias, however, is a constant. The noise from accelerometer can be analogously regarded as Gaussian white noise. Hence, it is obtained that $\eta(k) \sim N(\mu, Q(k))$, $v(k) \sim N(0, R(k))$. Introducing a new noise $w(k) \sim N(0, Q(k))$ which equals $\eta(k) - \mu$, Eq. 1 can be rewritten as

$$\begin{aligned} x(k) &= x(k-1) + \tau u(k-1) + \mu + w(k-1) \\ z(k) &= x(k) + v(k) \end{aligned} \tag{2}$$

So the problem is how to acquire the relatively optimal estimate from the two sensors, which can be treated as a problem on multi-sensor fusion. A conventional method to solve the problem on estimate is to employ the model-based Kalman filter, which is rigorous in theory but computationally complex. The complementary filter, complementary in the frequency domain, performs a frequency fusion of two sensors without considering the noise statistical characteristics. This approach is cost-efficient but there is no guarantee of optimality.

Furthermore, to facilitate the analysis, the process noise variance $Q(k)$ and measurement noise variance $R(k)$, which may change with time, are usually assumed to be constant in practice. The accuracy, however, deteriorates with the noise statistical characteristics changing. From Fig. 5 and Fig. 6, it can be known that the noise statistical characteristics are time-variant and peak at high-frequency motion.

IV. Fuzzy-Logic Based Adaptive Weighting Filter Design

To develop a cost-effective filtering approach characterized by computationally simple and theoretically optimal, Combining the advantages of Kalman filter coupled with complementary filter, the weighting filter is proposed. What is more, considering the time-variant property of noise statistical characteristics, a fuzzy-logic based regulator is introduced to adjust the weight adaptively, improving the filtering accuracy.

A. Weighting Filter

The Kalman filter estimates process states by using a form of feedback control: the filter estimates the process states at some time and then obtains feedback in the form of measurement. Hence, the equations for the Kalman filter fall into two groups: predictor equation and corrector equation. The predictor equation is responsible for projecting forward the current state and error covariance estimate to obtain the a priori estimates for the next time step. The corrector equation is responsible for the feedback, namely, for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate. The process is repeated with the previous a posteriori estimates used to predict the new a priori estimates. So the estimates of Kalman filter at every sampling time step can be written as.

$$\begin{aligned} \hat{x}(k|k) &= \tfrac{R(k)}{P(k|k-1)+R(k)}\hat{x}(k|k-1) + \tfrac{P(k|k-1)}{P(k|k-1)+R(k)}z(k) \\ \hat{x}(k|k-1) &= \hat{x}(k-1|k-1) + \tau u(k-1) + \mu \end{aligned} \tag{3}$$

where $\hat{x}(k|k)$ is the optimal estimate at sampling time step k, $\hat{x}(k|k-1)$ the a priori estimate at sampling time step $k-1$. Solving Riccati Equation the noise variance $P(k|k-1)$ is obtained as follows,

$$P(k|k-1) = \frac{P(k-1|k-2)R(k-1)}{P(k-1|k-2) + R(k-1)} + Q(k-1) \tag{4}$$

Under the assumption that $Q(k) = q^2$ and $R(k) = r^2$, it is obvious that $P(k|k-1)$ is non-negative and monotonically declining, consequently converging. Then

$$\lim_{k\to\infty} P(k|k-1) = \frac{q^2 + \sqrt{q^4 + 4q^2r^2}}{2} \tag{5}$$

So Eq. 3 can be rewritten as

$$\begin{aligned} &\hat{x}(k|k) = \\ &\alpha[\hat{x}(k-1|k-1) + \tau u(k-1) + \mu] + (1-\alpha)z(k) \end{aligned} \tag{6}$$

where $0 < \alpha < 1$ is the weight coefficient.

The filter seems to weight from the a priori estimate and measurement, to name the weighting filter. Noticing that this filter is recursive, the estimate is affected by the selection of initial value $\hat{x}(0|0)$. To study this problem, considering two different initial value $\hat{x}^{(1)}(0|0)$ and $\hat{x}^{(2)}(0|0)$, which result in two estimates $\hat{x}^{(1)}(k|k)$ and $\hat{x}^{(2)}(k|k)$ at sampling time k, respectively. Then

$$\begin{aligned} \hat{x}^{(1)}(k|k) &= \alpha[\hat{x}^{(1)}(0|0) + \tau u(k-1) + \mu] + (1-\alpha)z(k) \\ \hat{x}^{(1)}(k|k) &= \alpha[\hat{x}^{(2)}(0|0) + \tau u(k-1) + \mu] + (1-\alpha)z(k) \end{aligned} \tag{7}$$

The difference of the two estimates at sampling time k is.

$$\delta(k) = \hat{x}^{(1)}(t|t) - \hat{x}^{(2)}(t|t) = \alpha\delta(k-1) = \alpha^k\delta(0) \tag{8}$$

where $\delta(0) = \hat{x}^{(1)}(0|0) - \hat{x}^{(2)}(0|0)$. Due to that $0 < \alpha < 1$ the $\delta(k)$ will converge to 0 at $k \to \infty$. Hence, the filter is asymptotic stable that the initial value $\hat{x}(0|0)$ can be selected arbitrarily.

B. Fuzzy-Logic Based Regulator

It has been said in the previous section that the weight is determined by the variance of process and measurement noises. The weight is constant at noise statistical characteristics being time-invariant. Three experiments are done under $\alpha = 0.90$, 0.94, 0.99 and their results are shown in Fig. 7,

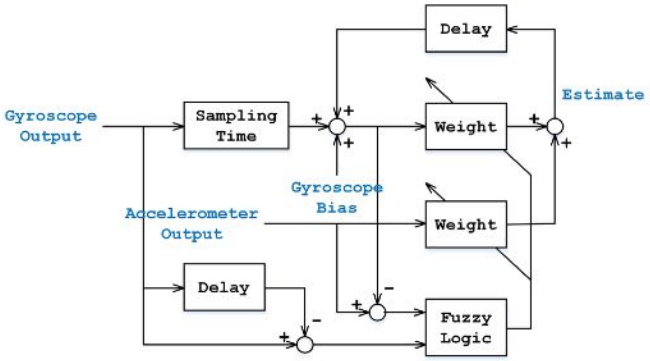


Fig. 3. Illustration of fuzzy-logic based adaptive weighting filter.

Fig. 8 and Fig. 9. The filtering accuracy is unacceptable due in large part to the weight which should be changed with time-variant noise statistical characteristics. Hence, it is necessary to develop a regulator to adjust the weight adaptively. substituting α by $\alpha(k)$ the time-variant filter is obtained.

$$\hat{x}(k|k) = \alpha(k)[\hat{x}(k-1|k-1) + \tau u(k-1) + \mu] + (1-\alpha(k))z(k) \quad (9)$$

Upon doing a lot of experiments, we find that the absolute of innovation $g(k) = |z(k) - \hat{x}(k|k-1)|$ and absolute of difference of gyroscope output $h(k) = |u(k) - u(k-1)|$ can be employed to determine the weight adaptively. The precise mathematical relationship, however, is hard to establish. Hence, fuzzy-logic is introduced to relate $g(k)$, $h(k)$ and $\alpha(k)$. This adaptive filter is illustrated in Fig. 3.

From the experiment results in Section 5, the fixed weight filter performs best at $\alpha = 0.94$. So we will develop the regulator based on $\alpha = 0.94$. It is represented mathematically as.

$$\alpha(k) = 0.94 + 0.07\gamma(k), \quad 0 \le \gamma(k) \le 1 \quad (10)$$

where the γ is determined by a set of fuzzy rules with the form as.

$$\text{if } h(k) \text{ is } A_i \text{ and } g(k) \text{ is } B_i, \text{ then } \gamma(k) \text{ equals } \gamma_i \quad (11)$$

The fuzzy rules in Eq. 11 is extracted experimentally based on the complementary characteristics of gyroscope and accelerometer. The high-frequency motion brings non-gravitational acceleration to result in a larger innovation $h(k)$. And the difference of gyroscope output $g(k)$, which is regarded as the rotation acceleration, is another reason for error. Hence, if either of them is relatively large, the fuzzy-logic based regulator judges that the $Q(k)$ and $R(k)$ are relatively large. Consequently the α gets large to weight more to gyroscope. The rules are shown in Table I.

TABLE I
RULES FOR γ REASONING

Rules	$g(k)$	$h(k)$	$\gamma(k)$
1	S	S	S
2	S	M	M
3	S	L	M
4	M	S	M
5	M	M	M
6	M	L	L
7	L	S	M
8	L	M	L
9	L	L	XL

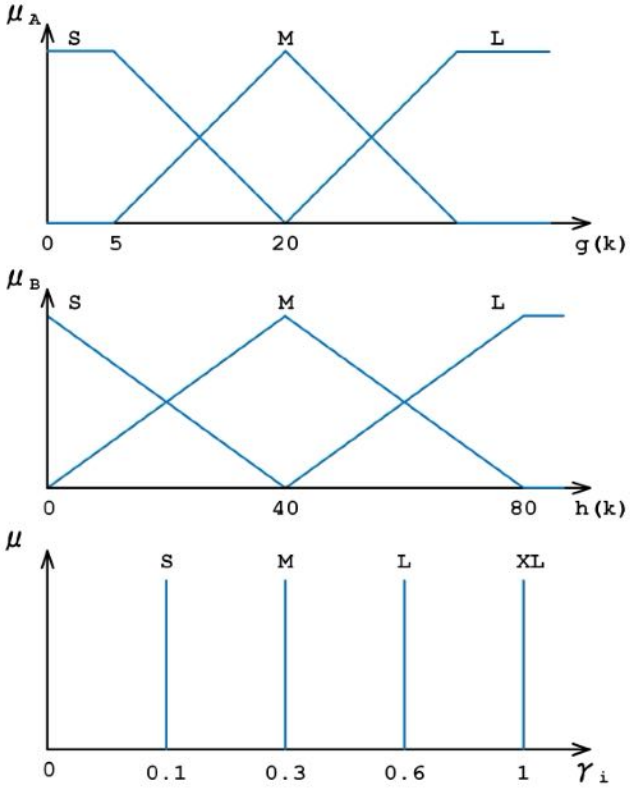


Fig. 4. Membership functions of $g(k)$, $h(k)$ and γ_i.

With a lot of experiments being conducted, the membership functions are obtained and shown in Fig. 4. In this figure, S represents for small, M for medium, L for large. It is noted that γ_i can also regarded as a fuzzy number with a singleton membership function, where S represents for small, M for medium, L for large, XL for extreme large. The real value of the ith rule in Eq. 11 is obtained by the product of the membership function values of $g(k)$ and $h(k)$ as.

$$\mu_i = \mu_{Ai}[g(k)] \times \mu_{Bi}[h(k)] \quad (12)$$

where μ_{Ai} is the membership function values of fuzzy set

A_i given a value of $g(k)$, and μ_{Bi} the membership function values of fuzzy set B_i given a value of $h(k)$.

Using the Centroid method, the γ can be obtained by defuzzification.

$$\gamma = \frac{\sum_{i=1}^{9} \mu_i \gamma_i}{\sum_{i=1}^{9} \mu_i} \tag{13}$$

where γ_i is the value of γ corresponding to μ_i for the ith rule. Once γ is obtained, the weight α can be calculated by Eq. 10.

V. Experiment Results

Fig. 5 and Fig. 6 show the measurement results using gyroscope or accelerometer separately. The red curves represent for reference, blue curves for the rotation angle estimate from the the integration of gyroscope output and accelerometer output, respectively. The green curves are the differences between reference and estimate, namely the noises. The black curve in Fig. 6 shows the accumulation error from the bias of gyroscope. It is easy to find that the noise from gyroscope can be analogously treated as Gaussian noise with a extreme small mean. This non-zero mean caused by bias, however, is a constant. The noise from accelerometer can be analogously regarded as Gaussian white noise.

Using fixed weighting filter to do experiments, three typical results at different weight $\alpha = 0.90, 0.94, 0.99$ are presented in Fig. 7, Fig. 8 and Fig. 9, respectively. Due to that the weight is too small to make the estimate inclined to accelerometer, experiment results shown in Fig. 7 resemble the results shown in Fig. 5. Experiment results shown in Fig. 9 appear drift for the reason that the larger weight makes the estimate rely more on gyroscope. Experiment results shown in Fig. 8 have the least error, but the error is large at high-frequency motion. This phenomenon is mainly caused by the variation of noise statistical characteristics.

The experiment results using fuzzy-logic based adaptive weighting filter is shown in Fig. 10. The red curves represent for reference, blue for estimate, green for error, black for the variation of $\alpha(k)$. Compared with the estimate using fixed weight filter at $\alpha = 0.94$ shown in Fig. 8, The error is reduced greatly less than ± 0.5. It can be found that $\alpha(k)$ gets large at high-frequency motion, weighting more to the gyroscope. Our design is validated to be cost-effective.

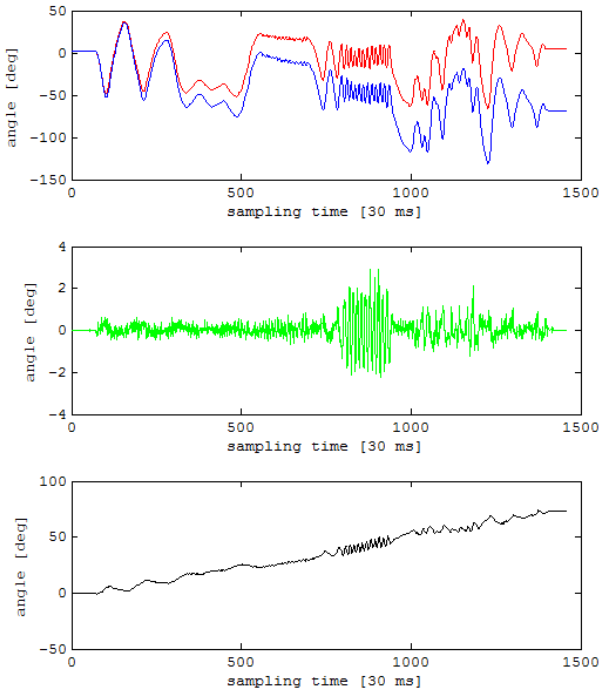


Fig. 5. Rotation angle measured by gyroscope.

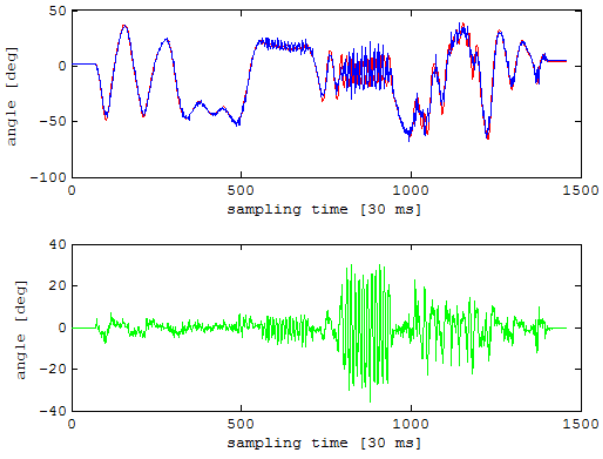


Fig. 6. Rotation angle measured by accelerometer.

VI. Conclusion

In this paper, a novel approach to the estimation of the rotation angle for strap-down inertial navigation system was proposed. Unlike the conventional filter, Kalman filter and complementary filter, the so-called weighting filter is trying to weight between the gyroscope and accelerometer. Our method is derived from the steady-state Kalman filter that the theoretical optimality could be guaranteed, but the computational complexity is not very high. That is to say, the weighting filter has the advantages of Kalman filter and complementary filter simultaneously. Considering the variation of noise statistical characteristics, a fuzzy-logic based regulator is developed to adjust the weight adaptively. The experiment results shows the effectiveness of our design.

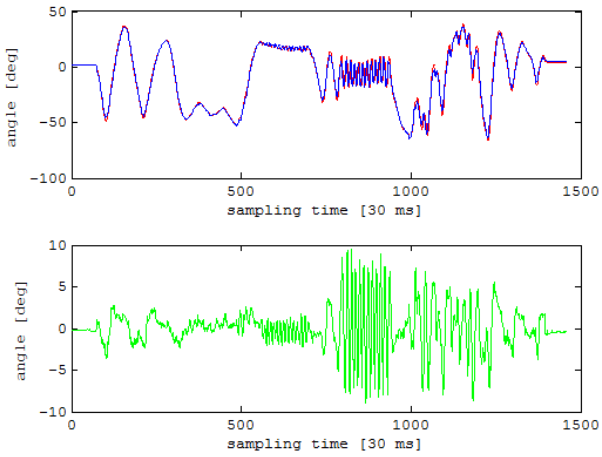


Fig. 7. Experiment at $\alpha = 0.90$.

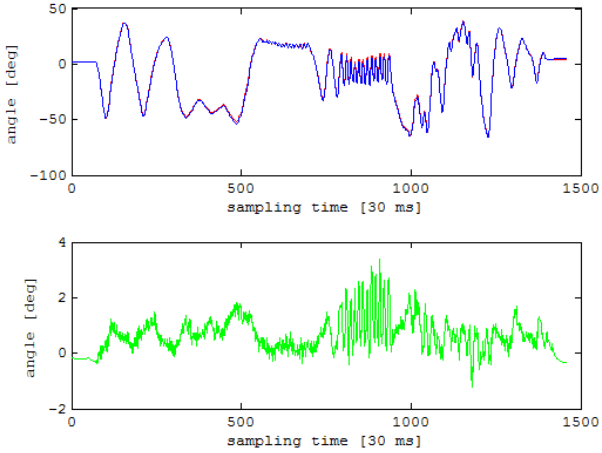


Fig. 8. Experiment at $\alpha = 0.94$.

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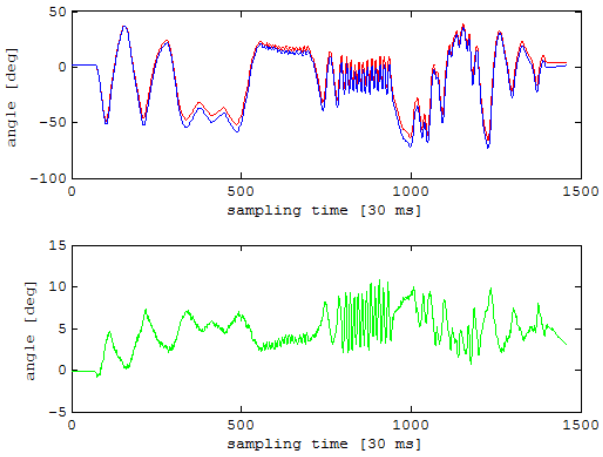


Fig. 9. Experiment at $\alpha = 0.99$.

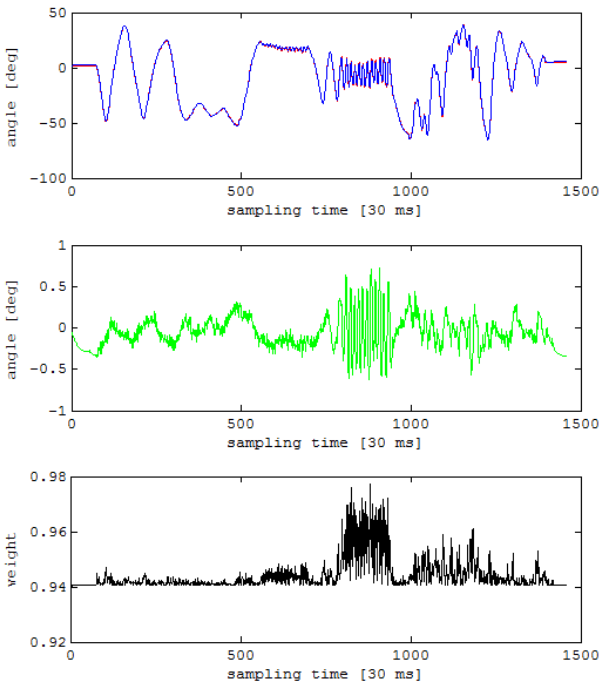


Fig. 10. Experiment result using fuzzy-logic base d weighting filter.

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